

## New dimensions for Randall-Sundrum phenomenology

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**ABSTRACT:** We consider a 6D extension of the Randall-Sundrum (RS) model, RS6, where the Standard Model (SM) gauge fields are allowed to propagate in an additional dimension, compactified on  $S^1$  or  $S^1/Z_2$ . In a minimal scenario, fermions propagate in the 5D RS subspace and their localization provides a model of flavor. New Kaluza-Klein (KK) states, corresponding to excitations of the gauge fields along the 6<sup>th</sup> dimension, appear near the TeV scale. The new gauge KK modes behave differently from those in the 5D warped models. These RS6 states have couplings with strong dependence on 5D field localization and, within the SM, only interact with heavy fermions and the Higgs sector, to a very good approximation. Thus, the collider phenomenology of the new gauge KK states sensitively depends on the 5D fermion geography. We briefly discuss inclusion of SM fermions in all 6 dimensions, as well as the possibility of going beyond 6D.

**KEYWORDS:** Phenomenology of Field Theories in Higher Dimensions.

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## 1. Introduction

The Randall-Sundrum (RS) model [1] has been extensively discussed as a resolution of the hierarchy between the  $\mathcal{O}(\text{TeV})$  weak scale in the Standard Model (SM) and the scale  $M_P \sim 10^{19}$  GeV of 4D gravity. The original model [1] was based on a slice of  $\text{AdS}_5$ , bounded by two 4D Minkowski branes. This model only addressed the weak- $M_P$  hierarchy, using the exponentially warped 5D spacetime, with 4D SM fields localized at the TeV (IR) brane and the 4D gravity localized near the Planck (UV) brane. It was later shown that extending the SM content to all 5 dimensions [2–5] still allows one to address the hierarchy, as long as the Higgs sector is localized near the TeV brane. An interesting consequence of this extension is that 5D fermion masses allow one to localize the zero modes of these fields along the 5<sup>th</sup> dimension [4] and provide a predictive model of flavor [4, 5].

Various extensions of the RS model have been considered in the literature. Much of the discussion has been concerned with expanding the bulk field content and extensions of 5D gauged symmetries, in order to enhance the agreement of the of the model with low energy data. In comparison, less attention has been devoted to extending the geometrical basis of warped models.

The RS model can be considered to be an effective theory, emerging from a string theoretic construction. Also, the AdS/CFT correspondence [6] has been very helpful in relating the geometric results in the RS picture to those arising from 4D strong dynamics [7, 8]. These theoretical aspects are generally contained in a larger picture with more than 5 dimensions. Viewed in this way, it is natural to consider adding additional dimensions to the RS geometry and studying their potential observable consequences. Some work along this direction can be found in refs. [9–12], where dimensions beyond the original 5D have been considered. However, these works, by and large, have concentrated on the gravitational sector and its phenomenology.

In this paper, we consider extending RS-type models with additional non-warped dimensions, where the gauge sector of the SM is also allowed to propagate in all dimensions. We adopt a minimal setup, where SM fermions are only allowed to travel in the original warped 5D spacetime, in order to address flavor physics, but some or all gauge fields are allowed to reside in 6 dimensions. We will consider an  $S^1$  or  $S^1/Z_2$  compactification for the 6<sup>th</sup> dimension and refer to this extended geometry as RS6. We concentrate on a 6 dimensional  $SU(3)_c$  sector and show that new Kaluza-Klein (KK) fields emerge at the TeV scale, with couplings very different from those that arise in the usual 5D picture [5, 13]. The couplings of these new modes to 5D fermion fields are shown to be exponentially sensitive to localization along the 5<sup>th</sup> dimension. We then begin to consider the collider phenomenology of these new KK modes at the LHC and their potential for discovery.

In the next section, we derive the KK equation of motion and the spectrum, for a gauge field in RS6. We also briefly discuss extensions to higher dimensional spheres. In section III, we derive the couplings of these KK modes to 5D fermions. In section IV, we discuss the LHC phenomenology of this model and outline its discovery prospects. We will also briefly discuss possible extensions of our minimal RS6 model to  $RS_n$ ,  $n > 6$ , as well as scenarios with fermions in more than 5 dimension. We will conclude in section V. The appendix provides some relevant expressions for the RS7 case with an  $S^2$  compactification.

## 2. KK spectrum and wavefunctions

Lets us consider the RS6 metric  $G_{MN}$ ,  $M, N = 0, 1, \dots, 5$ ;  $x_4 = r_c\phi, x_5 = R\theta$ , with an extra dimension compactified on  $S^1$ :

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2 - R^2 d\theta^2, \quad (2.1)$$

where, as usual  $\sigma(\phi) = kr_c|\phi|$ ,  $k$  is the scale of curvature and  $r_c$  is the radius of compactification of the  $AdS_5$  slice;  $\phi \in [-\pi, \pi]$  and a  $Z_2$  orbifolded 5<sup>th</sup> dimension is assumed. Here,  $R$  is the radius of  $S^1$  and  $\theta \in [0, 2\pi]$ ; in the absence of fine-tuning, it is natural to imagine that, e.g.,  $kR \sim 1$ . The choice of the 6D energy momentum tensor consistent with this background has been discussed in ref. [11], where the corresponding gravitational sector was studied.

The action for a 6D non-interacting gauge field is given by

$$S_A = -\frac{1}{4} \int r_c d\phi \int R d\theta \sqrt{-G} G^{AM} G^{BN} F_{AB} F_{MN}, \quad (2.2)$$

where  $G = \det(G_{MA})$  and  $F_{MN} = \partial_M A_N - \partial_N A_M$ . As is well-known, with 2 extra dimensions, in addition to the 4D gauge fields there is also a 4D tower of KK scalars that correspond to a combination of  $A_\phi$  and  $A_\theta$ . For example, in the case of  $SU(N)$ , this would correspond to a tower of massive adjoint scalars without a zero-mode. As is also well-known in the case of flat extra dimensions, such scalars can lead to their own interesting new physics[14]. Here, we will concentrate on the 4D vector modes  $A_\mu$ ,  $\mu = 0, 1, 2, 3$ , and set  $A_\phi = A_\theta = 0$ . With this choice, the action (2.2) yields

$$S_A = \int r_c d\phi \int R d\theta \left\{ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} \left[ \frac{1}{r_c^2} \partial_\phi (e^{-2\sigma} \partial_\phi A^\mu) A_\mu + e^{-2\sigma} \frac{1}{R^2} \partial_\theta^2 A^\mu A_\mu \right] \right\}. \quad (2.3)$$

The vector field  $A_\mu(x, \phi, \theta)$  can be expanded in KK modes

$$A_\mu(x, \phi, \theta) = \sum_{n,l} A_\mu^{(n,l)}(x) \frac{\chi^{(n,l)}(\phi)}{\sqrt{r_c}} \frac{\varphi^{(l)}(\theta)}{\sqrt{R}}. \quad (2.4)$$

The  $\theta$ -dependent wavefunction is given by

$$\varphi^{(l)}(\theta) = e^{il\theta} / \sqrt{2\pi} \quad (2.5)$$

in the case of  $S^1$  and

$$\varphi^{(l)}(\theta) = \begin{cases} 1/\sqrt{2\pi}, & l = 0 \\ \cos(l\theta)/\sqrt{\pi}, & l \neq 0 \end{cases} \quad (2.6)$$

for the orbifolded  $S^1/Z_2$  case. The wavefunctions obey the orthonormality conditions

$$\int d\phi \chi^{(m,l)} \chi^{(n,l)} = \delta_{mn} \quad (2.7)$$

and

$$\int d\theta \varphi^{(l)}(\theta) \varphi^{(l')}(\theta) = \delta_{ll'}. \quad (2.8)$$

Inserting the above KK expansion into the action (2.3), we find the following eigenvalue equation for the  $(n, l)$  mode of mass  $m_{nl}$

$$-\frac{1}{r_c^2} \frac{d}{d\phi} \left( e^{-2\sigma} \frac{d}{d\phi} \chi^{(n,l)}(\phi) \right) + e^{-2\sigma} \left( \frac{l}{R} \right)^2 \chi^{(n,l)}(\phi) = m_{nl}^2 \chi^{(n,l)}(\phi). \quad (2.9)$$

The above equation of motion corresponds to that for a 5D vector field of bulk mass  $l/R$ , in the RS background. The solutions are given by [3]

$$\chi^{(n,l)}(\phi) = \frac{e^\sigma}{N_{nl}} [J_\nu(z_{nl}) + \alpha_{nl} Y_\nu(z_{nl})], \quad (2.10)$$

where  $J_\nu$  and  $Y_\nu$  denote Bessel functions of order  $\nu$  where

$$\nu \equiv \sqrt{1 + \left( \frac{l}{kR} \right)^2}, \quad (2.11)$$

with  $z_{nl}(\phi) \equiv (m_{nl}/k)e^\sigma$ . We will define for simplicity the combination

$$\zeta_\nu(z_{nl}) \equiv J_\nu(z_{nl}) + \alpha_{nl} Y_\nu(z_{nl}). \quad (2.12)$$

We then impose the boundary conditions  $\partial_\phi \chi^{(n,l)}(\phi) = 0$  at  $\phi = 0, \pi$ , which yield

$$z_{nl} \zeta'_\nu + \zeta_\nu = 0. \quad (2.13)$$

Using eq. (2.7) and eq. (2.13), we find for the normalization

$$N_{nl} = \frac{e^{kr_c\pi}}{x_{nl} \sqrt{kr_c}} \sqrt{\zeta_\nu [z_{nl}^2 - (\nu^2 - 1)]} \Big|_{\varepsilon_{nl}^{x_{nl}}}, \quad (2.14)$$

where  $x_{nl} = z_{nl}(\pi)$  and  $\varepsilon_{nl} = z_{nl}(0)$ .

Eq. (2.13) evaluated at  $\phi = 0$  can be used to determine the coefficients  $\alpha_{nl}$ :

$$\alpha_{nl} = -\frac{J_\nu(\varepsilon_{nl}) + \varepsilon_{nl}J'_\nu(\varepsilon_{nl})}{Y_\nu(\varepsilon_{nl}) + \varepsilon_{nl}Y'_\nu(\varepsilon_{nl})}. \quad (2.15)$$

One can then easily show that  $\alpha_{nl} \sim \varepsilon_{nl}^{2\nu}$ . Since addressing the hierarchy implies  $\varepsilon_{nl} \sim 10^{-16}$ , for  $\nu > 1$ , one can safely ignore the part of the wavefunction  $\chi^{(n,l)}$  that is proportional to  $\alpha_{nl}$ . The masses of the KK modes corresponding to  $\chi^{(n,l)}$  are given by  $m_{nl} = x_{nl} k e^{-kr_c\pi}$ , where  $x_{nl}$  are the roots of the transcendental equation

$$J_\nu(x_{nl}) + x_{nl}J'_\nu(x_{nl}) = 0, \quad (2.16)$$

obtained from eq. (2.13) at  $\phi = \pi$ , ignoring terms proportional to  $\alpha_{nl}$ . In this approximation, we then find

$$\chi^{(n,l)}(\phi) \simeq \frac{e^\sigma}{N_{nl}} J_\nu(z_{nl}), \quad (2.17)$$

where

$$N_{nl} \simeq \frac{e^{kr_c\pi}}{\sqrt{kr_c}} \beta(x_{nl}, l) J_\nu(x_{nl}), \quad (2.18)$$

and

$$\beta(x_{nl}, l) \equiv \left[ 1 - \left( \frac{l}{kR x_{nl}} \right)^2 \right]^{1/2}. \quad (2.19)$$

For the purposes of this work, eqs. (2.16), (2.17), and (2.18) are very good approximations and will be used in what follows. Note that since  $\beta$  is a real quantity we must have  $x_{nl} > l/kR$  and thus states with  $l > 0$  do not have zero-modes.

One can similarly derive expressions for KK gauge fields from compactification on  $S^N$ , with  $N > 1$ . As an example, we display the wavefunctions for the case of  $S^2$  in the appendix. Much of what follows in the next section can then be applied to higher dimensional spherical compactifications, with rather straightforward modifications.

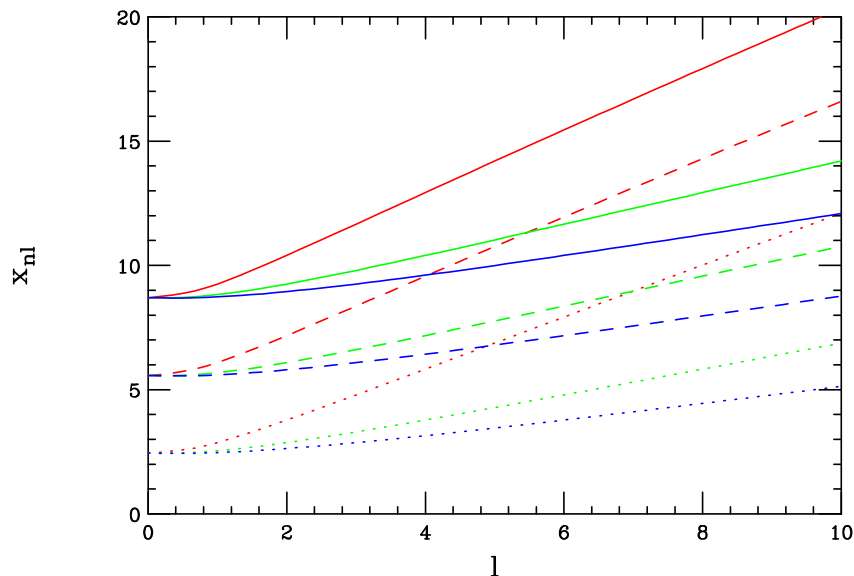
### 3. KK couplings in the minimal RS6 model

Here, we consider a minimal extension where gauge fields are allowed to propagate in all 6 dimensions and the SM fermions reside in the 5D RS subspace, in order to explain the flavor structure observed at low energies. Later we will discuss typical fermion ‘‘geographies’’ and their experimental consequences in the context of this minimal RS6 model. For simplicity we will assume that the SM fermions are localized at  $\theta = 0$ . To get some sense of the magnitude of the couplings to SM fermions, we will first consider two extreme cases, with a fermion localized at either the UV or the IR brane. At the UV brane,  $\phi = 0$ , and we have (for  $l > 0$ )

$$\chi^{(n,l)}(0) \sim \varepsilon_{nl}^{\nu+1}, \quad (3.1)$$

whereas at the IR brane,  $\phi = \pi$ , we have for all  $l$

$$\chi^{(n,l)}(\pi) \simeq \sqrt{kr_c}/\beta(x_{nl}, l). \quad (3.2)$$



**Figure 1:** The lowest lying roots,  $x_{nl}$ , assuming  $kR = 1$ (red), 2(green) or 3(blue) for  $n = 1$ (dots), 2(dashes) or 3(solid) as a function of  $l$ .

Using the zero mode wavefunction  $\chi^{(0,0)} = 1/(2\pi)$ , one can easily derive the relation

$$g_4 = \frac{g_6}{2\pi\sqrt{r_c R}}, \tag{3.3}$$

between the 4D and 6D gauge couplings,  $g_4$  and  $g_6$ , respectively. Using eq. (3.1), one then finds that the  $l \neq 0$  KK modes *exponentially decouple* from fermions at the UV brane. However, the couplings  $g_{nl}$ , for  $l \neq 0$ , to the IR brane fermions are given by

$$g_{nl}|_{\text{IR}} = \begin{cases} g_4\sqrt{2\pi kr_c}/\beta(x_{nl}, l), & S^1 \\ g_4\sqrt{4\pi kr_c}/\beta(x_{nl}, l), & S^1/Z_2, \end{cases} \tag{3.4}$$

where we have used eq. (3.2). Given that  $1/\beta(x_{nl}, l) > 1$  and  $kr_c \sim 10$ , we see that  $g_{nl} \gtrsim 8g_4$  at the IR brane is quite large. Note that this coupling is larger by factors of  $1/\beta(x_{nl}, l)$ , for  $S^1$ , and  $\sqrt{2}/\beta(x_{nl}, l)$ , for  $S^1/Z_2$ , than the corresponding coupling in the 5D RS model. For example, if we take  $kR = 1$ , we obtain  $x_{10} \simeq 2.45$ ,  $x_{11} \simeq 2.87$ , and  $1/\beta(x_{11}, 1) \simeq 1.07$ . Hence, the ratio of the mass of the lightest  $l \neq 0$  KK mode to the lightest KK mode is  $2.87/2.45 \simeq 1.17$  in this case. Figure 1 shows some of the lowest lying roots obtained from eq. 16, which determines the gauge KK masses, as a function of the value of  $l$ . These will be important when we study the possible LHC signatures in the next section.

Having studied the extreme UV/IR brane cases, let us consider the intermediate cases where the fermions are not confined to either brane, but have 5D profiles. For a zero mode fermion, the bulk profile is given by [4, 5]

$$f_0 = \frac{e^{-c\sigma}}{N_0}, \tag{3.5}$$

with the normalization

$$N_0 = \left[ \frac{e^{kr_c\pi(1-2c)} - 1}{kr_c(1/2 - c)} \right]^{1/2}. \quad (3.6)$$

Here,  $c > 1/2$  corresponds to UV localization (light fermions) and  $c < 1/2$  corresponds to IR localization (heavy fermions).

The coupling  $g_{nl;c}$  of the  $(n, l)$  modes to a 5D zero mode fermion is then given by

$$g_{nl;c} = \sqrt{2\pi} \int d\phi f_0^2 \chi^{(n,l)}, \quad (3.7)$$

for  $S^1$ , and the corresponding  $S^1/Z_2$  value is larger by  $\sqrt{2}$  for  $l \neq 0$ . For example, if we choose typical values  $c = 0.6$  for light fermions and  $c = 0.3$  for heavy fermions and set  $kR = 1$ , we then find  $g_{11;0.6} = 9 \times 10^{-4}g_4$  and  $g_{11;0.3} = 2.1g_4$ , for the  $S^1$  compactification. Thus, typically, we expect the  $l \neq 0$  modes to decouple from light fermions, to a good approximation. However, the coupling of these modes to light fermions has a strong dependence on the exact fermion localization in 5D.<sup>1</sup> This is in contrast to the original RS model, corresponding to  $l = 0$  here, where gauge KK couplings to light fermions are nearly universal and small, but not negligible; for example, taking  $c = 0.6$  one obtains instead  $g_{10;0.6} = 0.19g_4$ .

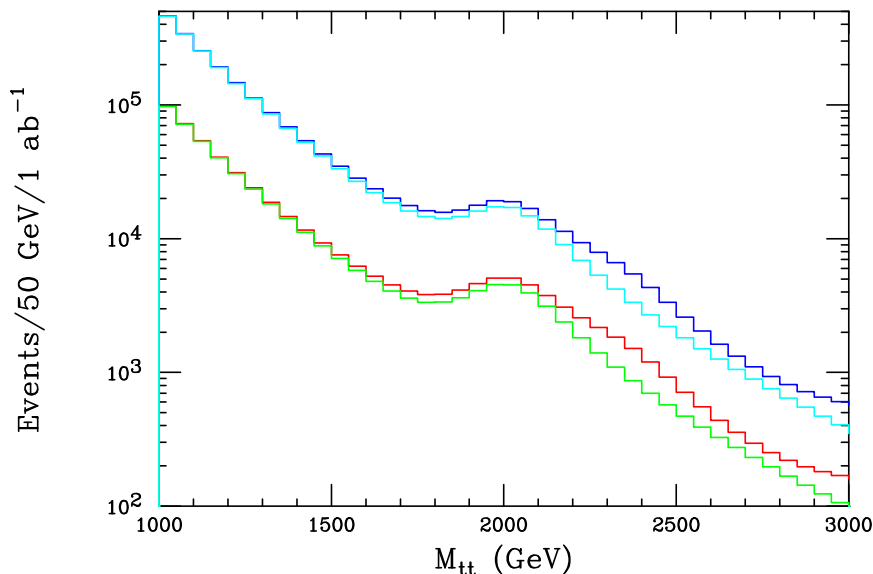
#### 4. LHC phenomenology

Here we address potential signals of the RS6 model at the LHC. Similar considerations can be applied to higher dimensional spherical compactifications  $S^N$ , with  $N > 1$ , keeping the SM fermions in the 5D subspace. For the 5D RS model, corresponding to  $l = 0$  modes, there is very little sensitivity in the gauge KK couplings to the UV localization ( $c > 1/2$ ) of light fermions and one can choose a typical value for  $c \gtrsim 1/2$  and obtain the universal KK coupling to light fermions of the SM. This allows for a relatively model independent assessment of the relevant collider phenomenology [15–17]. However, as we saw before, the couplings of the  $l \neq 0$  gauge KK modes are very sensitive to the 5D localization of the fermions. Thus, to study possible signatures of the RS6 model we must be more specific about the localization parameters for the important initial state fermions at colliders. In what follows we consider the simplest case of  $S^1$  compactification.

To make a comparison of our results with some of the existing literature easier, we adopt a 5D flavor model in which  $t_R$  is the most IR-localized SM fermion and couples to the lightest KK mode ( $l = 0$ ) with the strength  $g_4\sqrt{kr_c\pi}$ . Here, we will concentrate on KK gluons and hence  $g_4 = g_s$ , where  $g_s$  is the SU(3) coupling in the SM. This corresponds to  $c(t_R) = -0.6$ , in our convention. We choose the localization parameters close to those in realistic models [18], but we only attempt to capture the essential features of 4D flavor and not the details. To get the correct top mass, we then require  $c(Q_L^3) = 0.3$ , where  $Q_L^3$  denotes the third family quark doublet and we have assumed that the Higgs is on the IR brane.

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<sup>1</sup>Note that this may lead to additional flavor issues due to the exchanges of these states, but such a discussion is beyond the scope of the present paper.

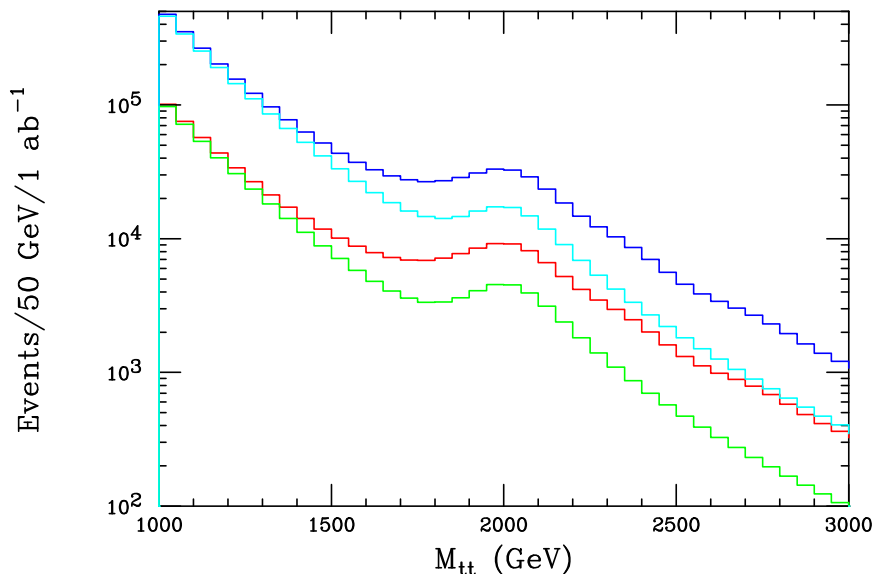


**Figure 2:** RS6 KK production for  $kR = 1$ , where the lightest state is at  $m_{10} = 2$  TeV. The upper and lower pairs of histograms correspond to cuts of  $|\eta| < 1, 1/2$ , respectively, on the final state tops. Both final state top quarks are also required to have  $p_T > 200$  GeV. In each pair, the upper histogram includes RS6 KK contributions up to states which are  $\sim 1.54$  times more massive than the lightest KK mode. The lower histogram in the pair represents the usual 5D RS scenario. An integrated luminosity of  $L = 1$   $\text{ab}^{-1}$  has been assumed.

This way, the dominant decay mode of the new states is into the channel  $t_R \bar{t}_R$ . However, in the following, we would like to address resonant production of the  $l \neq 0$  modes from  $q\bar{q}$  initial states. For very light quarks we saw above that these couplings were very small and so, e.g., conventional  $u\bar{u}$ ,  $d\bar{d}$  and  $s\bar{s}$  initial state partons lead to small cross sections. Given that  $b_L$  is IR localized with our choice of parameters, its coupling to the (11) state is  $\sim 2g_s$  and fairly large. Hence, it makes sense to examine whether one can use the  $b$ -content of the initial states for KK production. Here, we ignore higher order corrections to the flux of initial state  $b$ -quarks, which is a good approximation for our purposes [19].

Ignoring the rest of the quarks, for  $kR = 1$ , we found, however, that the small  $b$ -content of the initial state protons does not yield a significant signal for the  $l \neq 0$  modes, with an integrated luminosity  $L = 1$   $\text{ab}^{-1}$ . However, we have determined that the inclusion of the charm content, together with that of bottoms, of the proton plays an important role here. Choosing  $c(c_R) = 0.52$  for the singlet charm quark  $c_R$ , we find that its coupling to the lightest  $l \neq 0$  states is roughly  $0.07g_s$ . Even though this is not very large, it turns out that the much larger charm content of the proton compensates for it with the enhanced top quark coupling in the final state. In figure (2), we have presented the result for the case  $kR = 1$ , choosing the lightest state (10) to be at 2 TeV, and including the effects of coupling to  $c_R$  in the proton; here and for the rest of this discussion  $L = 1$   $\text{ab}^{-1}$  is assumed. Given the couplings above, very roughly, all of the KK states have width-to-mass ratios of  $\simeq 1/6$ . Note in addition that for the  $S^1$  compactification there are two degenerate states at each  $l \neq 0$  which is a significant source of cross section enhancement. The upper pair

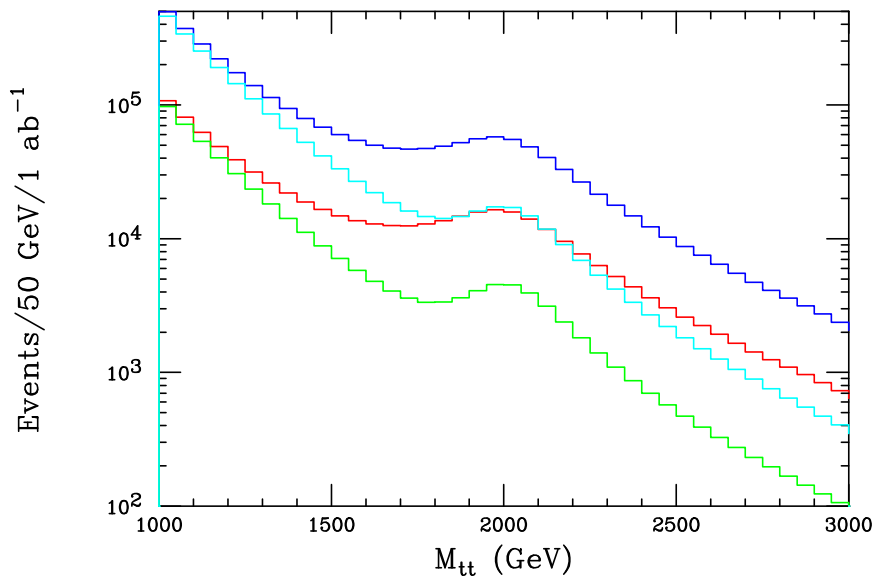




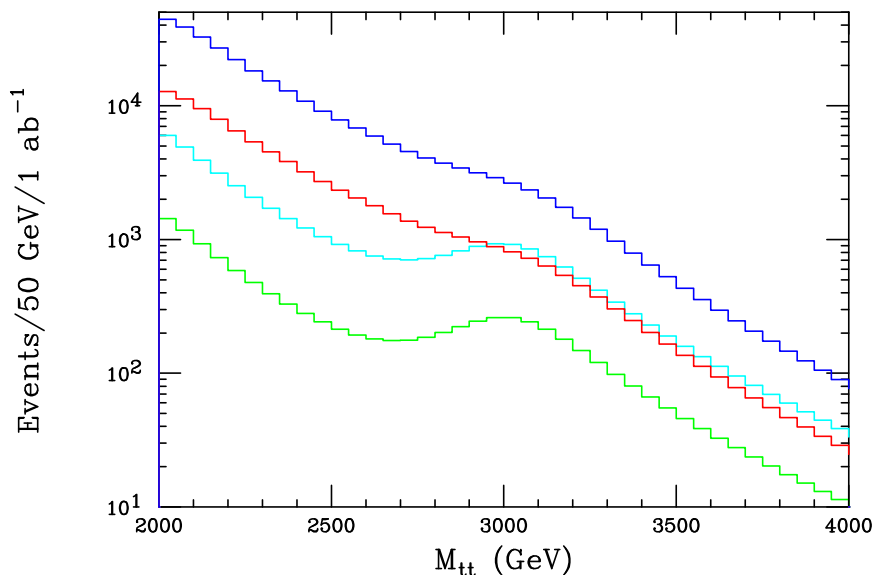
**Figure 3:** Same as the previous figure but now with  $kR = 2$ .

of curves correspond to the pseudo-rapidity cut  $|\eta| < 1$  and the lower one corresponds to  $|\eta| < 1/2$  on the top quarks in the final state. In each pair, the upper histogram includes the contributions of  $l \neq 0$  resonances, up to a state that is  $3.78/2.45 \simeq 1.54$  times heavier than the (10) KK mode. The lower histogram in the pair does not include any contribution from  $l \neq 0$  modes and corresponds to the usual 5D RS result. We see from the figure that when  $kR = 1$  there is no obviously clear signal for RS6 versus the usual RS. Note that we have not included here the effects of boosted top jets, branching fractions, efficiencies and detector resolutions that go into the actual extraction of the signal from the data. As these effects will make it only more difficult to distinguish the two cases, we conclude that the signal in this case is at best only very marginal.

One of the factors that made the signal in the previous case, with  $kR = 1$ , difficult to observe was that only the additional resonances for  $l = 1, 2$  were included in the mass range above the (10) state. Going to the case  $kR = 2$ , still a modest value and a reasonable choice, will increase the number of contributing resonances in our mass window and will significantly boost the signal. For this values of  $kR$ , the modes included in the mass range 2.0 – 3.1 TeV, considered in figure (2), correspond to  $l = 1, \dots, 4$ . The expected signal in this case is presented in figure (3), using the same cuts as before. We see that the signal is now much more pronounced and corresponds to a noticeably different line shape above the (10) resonance; this is due to the overlap of the contributions of multiple resonances which are each rather wide. Given the statistics inferred from the plot, we would expect a clear signal for RS6 in this case even when efficiencies etc are included. Figure (4) is the same as figure (3), but now for the case  $kR = 3$ . As expected, the RS6 signal is now significantly distinct from the RS case, since all states corresponding to  $l = 1, \dots, 6$  now contribute in the above mentioned mass interval. Clearly as  $kR$  increases further the deviation from the classic RS signature will only increase. Now that we see the pattern of change induced by



**Figure 4:** Same as the previous figure but now with  $kR = 3$ .



**Figure 5:** Same as the previous figure with  $kR = 3$  and  $m_{10} = 3$  TeV.

the  $l \neq 0$  states as we vary  $kR$ , it is clear that for values of  $kR < 1$  the new gauge KK states will be essentially invisible in the  $t\bar{t}$  channel.

Here we note that if RS-type models are to emerge at the TeV-scale, precision data strongly suggest that new symmetries need to be imposed on these models [20, 21]. Even then, the mass  $m_{10} = 2$  TeV of the lightest gluon KK state chosen for the above plots may not be consistent with current bounds on the RS model from precision data [22–26]. Hence, in figure (5), we present the  $kR = 3$  case with  $m_{10} = 3$  TeV, for which the model is in better agreement with the electroweak precision data (agreement with the flavor data [27]

typically requires further model building [28, 29]). Here, again, the plot suggests that the signal will be quite prominent and distinct from the usual RS expectation even after efficiencies etc are included. Note that in this case the peaking structure found in the 5D RS case is lost.

At this point we would like to discuss some future directions for going beyond the present work. In terms of collider signatures, a potentially interesting production channel may be radiation of the  $l \neq 0$  gauge KK states off a final state top quark, as the new KK modes couple to IR localized fields strongly. Also, within the setup studied here, for each 6D gauge group, there is a tower of scalar states in the adjoint representation. A suitable framework for 6D gauge-fixing in the RS6 model can allow one to identify the combination of the gauge field polarizations that correspond to this tower of physical scalars. We did not address this analysis here, and confined the scope of our project to the vector modes.

A possible extension of the RS6 model involves the inclusion of the fermions in all 6 dimensions. We did not consider this possibility within our minimal model, where 5D fermions are sufficient to address flavor physics. However, inclusion of the fermions in the 6D field content will require elimination of unwanted zero modes from the 4D effective theory, since 6D fermions come with  $\pm$  chiralities, each of which can be decomposed into both left- and right-handed 4D fermions [30].

Much of what we studied for the new vector KK modes in this work will go through with straightforward modifications for compactification on higher dimensional spheres. We have provided the relevant formalism in the case of  $S^2$ , in the appendix. We note that spheres do not allow for massless fermion zero modes [31, 32] and hence the appearance of 5D masses is expected to be a general consequence of compactification on these manifolds. We speculate that it may be possible to employ this feature in building warped models of flavor that use 5D masses for localization of fermions.

## 5. Conclusions

In this work, we considered extending the RS geometry to RS6 which includes an extra dimension compactified on  $S^1$  or  $S^1/Z_2$ . This is motivated by a UV completion of the RS model within string theory, where additional dimensions are present. We focused on a minimal model with a 6D gauge sector and 5D fermions, localized to explain SM flavor. We found the spectrum and wavefunctions of the new gauge KK modes, corresponding to excitations along the circle. These new modes have couplings that are more strongly sensitive to the 5D fermion geography than do the usual RS gauge KK modes. For values of the  $S^1$  radius that are somewhat large compared to the curvature of the slice of  $AdS_5$ , there are many new KK modes that are tightly spaced above the lightest RS KK mode. We discussed the potential for observation of these modes at the LHC and concluded that for reasonable choice of parameters, the usual RS resonance line shapes will be sufficiently modified to distinguish RS6 from the conventional 5D scenario. Future directions including other production channels, KK scalar phenomenology, inclusion of fermions in RS6, and higher dimensional compactifications were discussed briefly in the previous section.

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## A. Gauge KK wavefunctions for RS7 with $S^2$

We parameterize the metric as

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2 - R^2 (d\theta^2 + \sin^2 \theta d\omega^2), \quad (\text{A.1})$$

where we now have  $\theta \in [0, \pi]$  and  $\omega \in [0, 2\pi]$ .

Given the spherical symmetry of the compactification manifold, we choose the following KK expansion for the gauge field

$$A_\mu(x, \phi, \theta, \omega) = \sum_{n,l,m} A_\mu^{(n,l,m)}(x) \frac{\chi^{(n,l)}(\phi)}{\sqrt{r_c}} \frac{Y_l^m(\theta, \omega)}{R}, \quad (\text{A.2})$$

where the  $Y_l^m(\theta, \omega)$  are the spherical harmonics. We now have

$$\nu \equiv \sqrt{1 + \frac{l(l+1)}{(kR)^2}}. \quad (\text{A.3})$$

With this change the gauge KK masses as are given in the text above.

By spherical symmetry we may choose any point on the sphere to place the 5D fields. A particularly convenient choice is  $\theta = 0$ , for which we have

$$Y_l^m(0, \omega) = \sqrt{\frac{2l+1}{4\pi}} \delta_{m,0}. \quad (\text{A.4})$$

This coupling is independent of  $\omega$  and that for any  $l$  only allows  $m = 0$  states to couple. For the case when the 5D fields are localized at  $\theta = \pi$ , there will be an overall factor  $(-1)^l$ .

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